

LABORATOIRE D'ANALYSE ET DE MATHÉMATIQUES APPLIQUÉES - CNRS UMR 8050

## Franco-Dutch Workshop: Stat-Mech in Créteil 2023

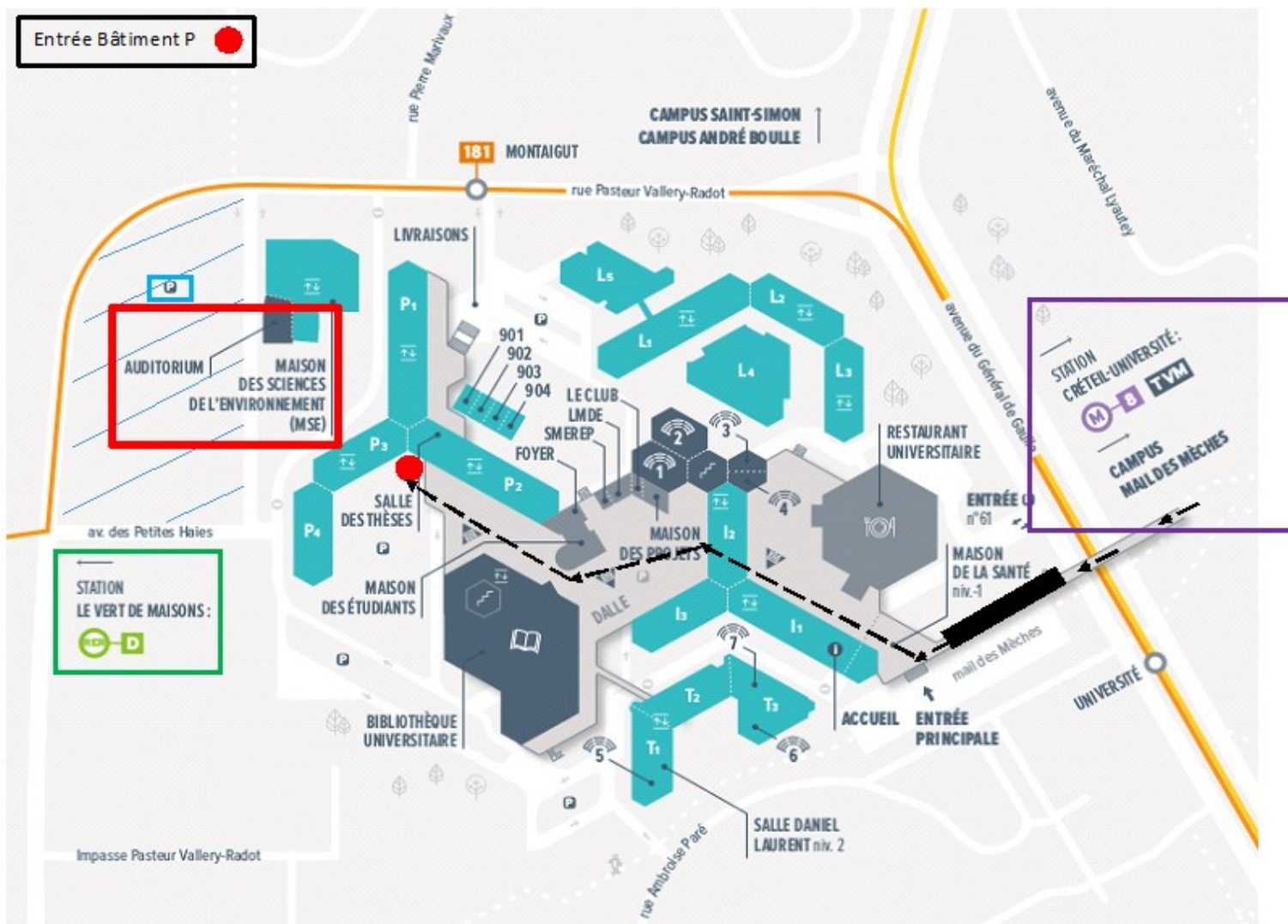
*One-day conference in Statistical Mechanics –  
June 13, 2023, Créteil*

**Localization (see Map):**

*LAMA-UPEC (P2/P3) & MSE (Amphi 8), Campus Centre*

*Auditorium de la Maison des Sciences et de l'Environnement (Amphi 8, MSE)*

**Registration:** [arnaud.le-ny@u-pec.fr](mailto:arnaud.le-ny@u-pec.fr) or [sonia.boufala@u-pec.fr](mailto:sonia.boufala@u-pec.fr)



## Preliminary Schedule

9h00-9h30	<i>Welcome coffee and tea</i>
9h30-10h15	<b>Matteo D'Achille</b>
	From local to global specifications: the spin-flop transition in XY model on $Z^2$ under decimation
10h15-10h30	<i>Coffee break and discussions</i>
10h30-11h15	<b>Aurélia Deshayes</b>
	Tug of war games in random graphs
11h15-12h00	<b>Irene Ayuso Ventura</b>
	Random boundary conditions for Ising models on Branching processes
12h-13h30	<i>Lunch (Brasserie du Palais, CC du Palais – “De Justice”)</i>
13h30-14h15	<b>Nadia Belmabrouk</b>
	Hawkes Processes with long-range interactions
14h15-14h30	<i>Coffee break and discussions</i>
14h30-15h15	<b>Eric Ossami Endo</b>
	CLT and Local CLT for long-range models at high temperatures
15h15-16h00	<b>Loren Coquille</b>
	Extremal Decomposition of free states on trees
16h00-16h15	<i>Coffee break and discussions</i>
16h15-17h00	<b>Christof Külske</b>
	Non-singleton localization at low temperature for gradient models on trees

## Titles and Abstracts

### Talk 1: Matteo D'Achille

#### From local to global specifications: the spin-flop transition in the XY model on $Z^2$ under decimation

**ABSTRACT:** The XY model is a widely studied model in statistical mechanics that describes the behavior of magnetic systems. In this model, each spin on a lattice is characterized by an angle that measures its orientation with respect to a fixed reference direction. The XY model on  $Z^2$  with nearest-neighbors ferromagnetic interactions famously exhibits algebraic decay of the two-point correlation function at low temperatures, as proven by Fröhlich--Spencer about 40 years ago. Therefore, a pertinent question is how the Gibbs measure of the XY model transforms under the action of renormalisation group transformations, such as decimation, at different temperatures.

In this talk, I will first discuss non-Gibbsianness of decimated measures at low temperatures for a large class of models and ferromagnetic interactions, including the nearest-neighbors XY model on  $Z^2$ . The basic ingredients are equivalence of boundary conditions and a recent extension to vector spins of the notion of *global specification*. The latter, originally due to Fernandez-Pfister for Ising spins, crucially allows conditioning on infinite subgraphs of the lattice.

Next, I will discuss how the decimated Gibbs measure, which does not display spontaneous symmetry breaking due to the Mermin-Wagner ban, still can become non-Gibbsian, due to the occurrence of a spin-flop transition obtained by constraining the decimated spins in a suitably chosen doubly alternating (bad) configuration.

Talk based on two papers in collaboration with Aernout van Enter and Arnaud Le Ny.

*Keywords: XY model, generalized Gibbs measure, equivalence of boundary conditions, global specification, spin-flop transition*

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### Talk 2: Aurélia Deshayes

#### Tug of war games in random graphs

**ABSTRACT:** It is well known that some partial differential equations, such as those involving the Laplace operator, can be interpreted using probabilistic objects such as random walks or Brownian motion. In a more general framework, the solutions of some nonlinear problems can be approximated by means of values of two-player games (with randomness).

In this talk, we will give an overview of these relationships at the interface between probability theory and PDEs and focus on tug-of-war games played on a random board, such as a geometric random graph.

This talk is based on a work in progress in collaboration with Nicolás Frevenza, Alfredo Miranda y Julio Rossi.

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### Talk 3: Irene Ayuso Ventura

#### Random boundary conditions on the Ising model on Branching Processes

**ABSTRACT:** Motivated by the study of the Ising model on tree-like random graphs with random boundary conditions, we define a random artificial boundary for a supercritical branching process. We show that the phase transition for the root magnetisation on the Ising model with this type of boundary is equivalent to that of the "+"-boundary conditions for a certain inhomogeneous tree model: the Pruned tree. We introduce the model and show criteria for the root magnetisation in terms of the 3-capacity of the Pruned tree.

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### Talk 4: Nadia Belmabrouk

#### Hawkes Processes on lattices with long-range interactions

**ABSTRACT:** We present new findings related to Hawkes process on infinite networks, which generalize the previous results of Delattre *et al.* (2016) from nearest neighbour's framework to a long-range interaction setting. Our analysis focuses on two regimes: the subcritical regime, where jump propagation is linear and the supercritical regime, which is characterized by exponential jump propagation.

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### Talk 5: Eric Ossami Endo

#### CLT and Local CLT for long-range models at high temperatures

**ABSTRACT:** The Local Central Limit Theorem is a result that provides a more refined result than the Central Limit Theorem. Both of them were vastly studied for random variables generated by models coming from Statistical Mechanics. Several papers such as [CCT,DG,DT] proved that, under certain conditions for the potentials, if the Central Limit Theorem holds for random fields defined on the lattice, then the Local Central Limit Theorem will hold as well. Considering a sufficiently high-temperature regime, we prove in [EM] that for a sequence of Gibbs measures with long-range, absolutely summable potentials on the lattice with spins taking lattice distributed values, for which the Integral Central Limit Theorem is satisfied, then the Local Central Limit Theorem also holds for that sequence. Our result complements [CCT], where some families of absolutely summable long-range potentials that fail the condition in [CCT] still satisfy the result at sufficiently high temperatures.

*References:*

- [CCT] M. Campanino, D. Capocaccia, and B. Tirozzi. The local central limit theorem for a Gibbs random field. *Comm. Math. Phys.* 70(2): 125–132, 1979.
- [DG] G. Del Grosso. On the local central limit theorem for Gibbs processes. *Comm. Math. Phys.* 37(2): 141–160, 1974.
- [DT] R.L. Dobrushin, and B. Tirozzi. The central limit theorem and the problem of equivalence of ensembles. *Comm. Math. Phys.* 54(2): 173–192, 1977.
- [EM] E.O. Endo, and V. Margarint. Local Central Limit Theorem for Long-Range Two-Body Potentials at Sufficiently High Temperatures. *J. Stat. Phys.* 189(34), 2022.
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## Talk 6: Loren Coquille

### Extremal decomposition of free states on trees

**ABSTRACT:** At very low temperature, the free state of the Ising model on a regular tree is not extremal. Moreover, its extremal decomposition is a continuous measure, supported on (uncountably many) inhomogeneous extremal states, which have some kind of « glassy » feature. I will present a proof of this result, which provides explicit concentration bounds on “branch overlaps”, playing the role of an order parameter for typical extremals. The proof extends quite generally to ferromagnetic finite-spin models, even slightly asymmetric (i.e. where small inhomogeneous field terms are added), which shows that the above behaviour is generic on regular trees.

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## Talk 7: Christof Külske

### Non-singleton localization at low temperature for gradient models on trees

**ABSTRACT:** We consider gradient models on regular trees. These are integer-valued spin-models, where the spins interact via a height-shift invariant interaction between nearest neighbors on the tree. We construct spatially homogeneous infinite-volume states, which localize on arbitrary given finite subsets  $A$  of the local state space. This is a new phenomenon, which we show to be robust via a new fixed point method. We also discuss related existence results for delocalized states, and spatially inhomogeneous states.

*References:*

1. A. Abbondandolo, F. Henning, C. Külske, P. Majer. *Infinite-volume states with irreducible localization sets for gradient models on trees.* ArXiv: 2302.05398
2. L. Coquille, C. Külske, A. Le Ny, *Extremal inhomogeneous Gibbs states for SOS-models and finite-spin models on trees.* *J. Stat. Phys.* **190** (2023), no. 4, Paper No. 71.