Analytic solutions of the 2D Euler equations with vortex layer initial data

M.Sammartino
Università degli Studi di Palermo

Abstract

In this talk we shall consider the 2D Euler equations

$$\begin{aligned} \partial_t \omega + \mathbf{u} \cdot \nabla \omega = 0 \\ u = \nabla^{\perp} \Delta^{-1} \omega \\ \omega(x, t = 0) = \omega_0(x) \end{aligned}$$

with initial data concentrated on a small set and satisfying the bound

$$\int |\omega_0| dx \leq c.$$

A significant instance of this configuration is when the vorticity $\omega = O(\epsilon^{-1})$ is distributed close to a curve $y = \phi_0(x)$ and decays away from it on a scale $O(\epsilon)$, being $\epsilon > 0$ a small parameter.

For analytic data and for a short time, we shall see that, in the limit $\epsilon \to 0$, the solutions do not develop oscillations or concentrations, and that the vorticity remains supported close to a curve $y = \phi(x, t)$ whose dynamics is ruled by the Birkhoff-Rott equation. No hypothesis on the sign of the vorticity is imposed.

We shall also present some numerical work highlighting the role of the Birkhoff-Rott singularity in the roll-up process of the vortex layer.

This is joint work with R.Caflisch and M.C.Lombardo and with F.Gargano and V.Sciacca.